

## Research Article

# Fixed-Time Feedback Control of the Hydraulic Turbine Governing System

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Dealing with convergence time and dealing with steady state are two of the most challenging problems in the field of stability of the hydraulic turbine governing system. In this paper, we solve these two challenging problems by designing fixed-time feedback controllers. The design of the controllers is based on the fixed-time theory and backstepping method. Compared with the existing controllers for fixed-time, finite-time, and other techniques, the designed controllers make the maximum convergence time of the system unaffected by the initial state. The convergence time is also shorter. Meanwhile, they are continuous and do not include any sign function, and hence, the chattering phenomenon in most of the existing results is overcome via nonchattering control. In addition, they give the system better stability and robustness to disturbances. Finally, the numerical simulation results in this paper will contribute to a better understanding of the effectiveness and superiority of the proposed controllers.

## 1. Introduction

Hydropower is a renewable energy source and it has high potential for economically viable utilization [1]. The past few decades have witnessed the rapid development of hydropower energy, which supplements or often replaces decommissioned coal-fired power plants [2]. The system of such a powerful hydropower plant including penstock systems, water turbines, generators, regulators, and loads, is so complex that it is difficult to control [3]. However, the stability of a hydropower system plays an important role in the stability of the whole power system and the plants themselves [4]. The hydraulic turbine governing system is one of the most important parts of a hydropower plant, which plays key roles in maintaining safety, stability, and economical operation of the hydropower plant [5]. It is also a multiparameter, high-dimension complex system with nonlinear, time-variant, and nonminimum phase characters [6]. Therefore, the study of the hydraulic turbine governing system is of great importance. In this regard, a special kind of nonlinear control technique, which combines the fixed-

time control theory, feedback control theory, and backstepping method, is introduced into the controller design of the hydraulic turbine governing system in this paper.

In the past decades, many advanced control techniques have been applied to the controller design of the hydraulic turbine governing system, such as PID control [7, 8], sliding-mode control [9], nonlinear control [10–12], fuzzy control [13], fault tolerant control [14], predictive control [15, 16], finite-time control [17], and fixed-time control [18–20]. These control methods have important theoretical and practical significance for the control of a hydraulic turbine governing system, but they also have some problems in overcoming the long-term operation of the system in real-time applications. For instance, the nonlinear nature of a water turbine and the constantly varying load makes the gain schedule of the PID controller difficult to design, limiting the operating range. Sliding-mode control is a quick and powerful variable-structure approach governing the nonlinear system via a predefined switch sliding surface. However, a common drawback of the conventional sliding-mode control is the chattering phenomenon, due to the use

of a discontinuous sign function. The nonlinear control is targeted, and each nonlinear control strategy is only suitable for solving some special nonlinear system control problems. Fuzzy control is difficult to adapt to the requirements of a large-scale adjustment, and it needs constant adjustment of the control rules and parameters. The convergence time of finite-time control is heavily dependent on the initial values of considered systems, which makes the maximum convergence time not fixed for different initial values. Moreover, not all the initial values of the systems are available in practice. These drawbacks prohibit the practical application of finite-time control [21]. In order to overcome the shortcomings of finite-time control, fixed-time control is proposed in [22]. The convergence time of fixed-time control can be completely determined by the parameters of a closed-loop system. Based on the advantages of fixed-time control, this control strategy has been applied to some systems [23–29]. It is worth mentioning that the sign function is indispensable for the controllers in [23–29]. It is well known that the sign function always introduces the chattering phenomenon to the system state and control signal, which damages equipment and induces undesirable effects [30]. Therefore, how to design continuous controllers without a sign function to make the system achieve fixed-time stability more rapidly has become the goal of this paper.

Motivated by the above discussions, to ensure the stability of a hydraulic turbine governing system, a set of ingenious controllers combining fixed-time control and feedback control is proposed. Compared with all the above control strategies, the designed fixed-time feedback controllers not only make the system steady in a fixed time, but also eliminate the chattering phenomenon of the system. In addition, they make the system stabilize faster and be more robust to disturbances. The rest of this paper is organized as follows. In Section 2, the model of the hydraulic turbine governing system is presented and some necessary definitions and lemmas are given. In Section 3, fixed-time feedback controllers are designed. Numerical simulations and theoretical analyses are presented to show the effectiveness of the controllers in Section 4. Finally, in Section 5, conclusions are given.

## 2. Preliminaries

**2.1. Model of Hydraulic Turbine Governing System.** The mathematical model of the hydraulic turbine governing system with the elastic water hammer has been widely studied in [12, 15, 31]. In this paper, the focus is to use an advanced control strategy to give the system better quality. Therefore, we use the nonlinear model of the hydraulic turbine governing system which is based on the previous work as follows:

$$\begin{aligned}\dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -a_0x_1 - a_1x_2 - a_2x_3 + y, \\ \dot{\delta} &= \omega_0\omega,\end{aligned}$$

$$\begin{aligned}\dot{\omega} &= \frac{1}{T_{ab}}(m_t - D\omega - P_e), \\ \dot{y} &= \frac{y}{T_y},\end{aligned}\quad (1)$$

where

$$\begin{aligned}P_e &= \frac{E'_q V_s}{x_{d\Sigma}} \sin(\delta + \delta_0) + \frac{v_s^2 x_{d\Sigma}' - x_{q\Sigma}}{2 x_{d\Sigma}' - x_{q\Sigma}} \sin(2\delta + 2\delta_0), \\ m_t &= b_3 y + (b_0 - a_0 b_3)x_1 + (b_1 - a_1 b_3)x_2 + (b_2 - a_2 b_3)x_3, \\ x_{d\Sigma}' &= x_{d'} + x_T + \frac{1}{2}x_L, \\ x_{q\Sigma} &= x_q + x_T + \frac{1}{2}x_L, \\ b_0 &= \frac{24e_y}{e_{qh}h_w T_r^3}, \\ b_1 &= \frac{24ee_y}{e_{qh}T_r^2}, \\ b_2 &= \frac{3e_y}{e_{qh}h_w T_r}, \\ b_3 &= \frac{ee_y}{e_{qh}}, \\ a_0 &= \frac{24}{e_{qh}h_w T_r^3}, \\ a_1 &= \frac{24}{T_r^2}, \\ a_2 &= \frac{3}{e_{qh}h_w T_r}.\end{aligned}\quad (2)$$

$x_1, x_2,$  and  $x_3$  denote intermediate state variables,  $\delta$  denotes the deviation of the generator rotor angle,  $\omega$  denotes the deviation of the generator rotor speed,  $y$  denotes the deviation of the guide vane opening. The meanings of the other symbols in the system are shown in the Nomenclature.

**2.2. Definition and Lemmas.** In this section, for the convenience of analysis, we present the basic definition of fixed-time stability first and then introduce some useful lemmas which are necessary for controller design.

*Definition 1* (see [22]). Consider the following nonlinear dynamic system.

$$\dot{x} = f(x), \quad (3)$$

where  $x \in R^n$  is the system state and  $f$  is a smooth nonlinear function. If there exists a fixed convergence time  $T_0$ , which is independent of the initial condition and satisfies

$$\lim_{t \rightarrow T_0} \|x(t)\| = 0, \quad (4)$$

then this nonlinear dynamic system is said to be fixed-time stable.

**Lemma 1** (see [22]). *Consider the following. If there exists a continuous radically unbounded function  $V : \mathbb{R}^n \rightarrow \mathbb{R}_+ \cup \{0\}$ , such that*

- (1)  $V(x) = 0 \Leftrightarrow x = 0$ ,
- (2) for some  $\alpha, \beta, \phi = 1 - 1/2\gamma, \varphi = 1 + 1/2\gamma$ , and  $\gamma > 1$ , any solution  $x(t)$  satisfied the inequality

$$D^* V(x(t)) \leq -\alpha V^\phi(x(t)) - \beta V^\varphi(x(t)), \quad (5)$$

where  $D^* V(x(t))$  denotes the upper right-hand derivative of the function  $V(x(t))$ , then the origin is globally fixed-time stable and the following estimate holds

$$T(x_0) \leq T_{\max} = \frac{\pi\gamma}{\sqrt{\alpha\beta}}, \quad \forall x_0 \in \mathbb{R}^n. \quad (6)$$

**Lemma 2** (see [32]). *If  $x_1, x_2, \dots, x_n \geq 0$ , Then, the following two inequalities hold.*

$$\begin{aligned} \sum_{i=1}^n x_i^\eta &\geq \left( \sum_{i=1}^n x_i \right)^\eta, \quad 0 < \eta \leq 1, \\ \sum_{i=1}^n x_i^\theta &\geq n^{1-\theta} \left( \sum_{i=1}^n x_i \right)^\theta, \quad \theta > 1. \end{aligned} \quad (7)$$

### 3. Fixed-Time Feedback Controllers Design

From (1), it is known that when the system is running to the point  $P(0,0,0,0,0,0)$ , the system returns to its original state of stability. In order to quickly stabilize the system to the equilibrium point  $P$ , the fixed-time controllers  $u_\omega$  and  $u_y$  are added to the fifth subsystem and sixth subsystem of the system (1), respectively, and the controlled system can be described as follows:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -a_0 x_1 - a_1 x_2 - a_2 x_3 + y, \\ \dot{\delta} &= \omega_0 \omega, \\ \dot{\omega} &= \frac{1}{T_{ab}} (m_t - D\omega - P_e) + u_\omega, \\ \dot{y} &= \frac{1}{T_y} (-y + u_y). \end{aligned} \quad (8)$$

For the uncertain system (1), in order to make the hydraulic turbine governing system stabilize in fixed time, we design the controllers which are based on the fixed-time

control theory, feedback control theory, and backstepping method as follows:

$$\begin{aligned} u_y &= -k_1 y - k_2 y^{m/n} - k_2 y^{p/q}, \\ u_\omega &= -\omega_0 \delta - \frac{m_t - D\omega - P_e}{T_{ab}} - k_3 \omega_0 \omega - \frac{mk_4 \omega_0 \omega}{n} \delta^{(m-n)/n} \\ &\quad - \frac{pk_4 \omega_0 \omega}{q} \delta^{(p-q)/q} - k_3 \left( \omega + k_3 \delta + k_4 \delta^{m/n} + k_4 \delta^{p/q} \right) \\ &\quad - k_5 \left( \omega + k_3 \delta + k_4 \delta^{m/n} + k_4 \delta^{p/q} \right)^{m/n} \\ &\quad - k_5 \left( \omega + k_3 \delta + k_4 \delta^{m/n} + k_4 \delta^{p/q} \right)^{p/q}, \end{aligned} \quad (9)$$

where  $k_1 > 0, k_2 > 0, k_3 > 0, k_4 > 0$ , and  $k_5 > 0$ , and  $m, n, p$ , and  $q$  are positive odd integers satisfying  $m < n$  and  $p > q$ ; thus, the hydraulic turbine governing system is fixed-time stable.

Here, we will give the design process of the fixed-time feedback controllers  $u_y$  and  $u_\omega$  of the hydraulic turbine governing system step by step. Consider the Lyapunov function:

$$V_1(t) = \frac{y^2}{2}. \quad (10)$$

Differentiating  $V_1(t)$  along the solution of (8) gives the following:

$$\dot{V}_1(t) = y\dot{y} = \frac{y}{T_y} (-y + u_y). \quad (11)$$

According to (11), in order to achieve fixed-time stability for the  $V_1(t)$ , the actual fixed-time feedback control law is obtained as follows:

$$u_y = -k_1 y - k_2 y^{m/n} - k_2 y^{p/q}, \quad (12)$$

where  $-k_2 y^{m/n} - k_2 y^{p/q}$  is to ensure the fixed-time stability of  $V_1(t)$  and  $-k_1 y$  is to speed up the stability of  $V_1(t)$ .

To bring (12) into  $V_1(t)$ , the following can be obtained:

$$\begin{aligned} \dot{V}_1(t) &= \frac{y}{T_y} (-y - k_1 y - k_2 y^{m/n} - k_2 y^{p/q}) \\ &= -\frac{1}{T_y} \left( y^2 + k_1 y^2 + k_2 y^{(m+n)/n} + k_2 y^{(p+q)/q} \right) \\ &\leq -\frac{k_2}{T_y} \left[ (y^2)^{(m+n)/2n} + (y^2)^{(p+q)/2q} \right] \\ &= -\frac{k_2}{T_y} \left[ \left( \frac{y^2}{2} \right)^{(m+n)/2n} \cdot \left( \frac{1}{2} \right)^{-(m+n)/2n} \right. \\ &\quad \left. + \left( \frac{y^2}{2} \right)^{p+q/2q} \cdot \left( \frac{1}{2} \right)^{-p+q/2q} \right] \\ &= -\frac{k_2}{T_y} \cdot 2^{(m+n)/2n} V_1^{(m+n)/2n}(t) - \frac{k_2}{T_y} \cdot 2^{p+q/2q} V_1^{p+q/2q}(t), \end{aligned} \quad (13)$$

where  $\alpha_1 = k_2/T_y \cdot 2^{(m+n)/2n}$ ,  $\beta_1 = k_2/T_y \cdot 2^{(p+q)/2q}$ ,  $\phi_1 = (m+n)/2n$  and  $\varphi_1 = (p+q)/2q$ . According to Lemma 1, we know that the sixth subsystem of (8) is stable in fixed time

$$t_1 = \frac{T_y \pi q n}{k_2 (pn - mq) \cdot 2^{(qm+pn-2qn)/4qn}}, \quad (14)$$

which means that the system state variable  $y$  satisfies the following relation  $y = 0$  when  $t \geq t_1$ . And

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= -a_0 x_1 - a_1 x_2 - a_2 x_3, \\ \dot{\delta} &= \omega_0 \omega, \\ \dot{\omega} &= \frac{1}{T_{ab}} (m_t - D\omega - P_e) + u_\omega. \end{aligned} \quad (15)$$

For the system (15), the desired controllers  $u_\omega$  can be worked out by the following steps.

Step 1. Define the tracking errors as follows:  $e_1 = \delta$ ,  $e_2 = \omega - \omega_d$ , in which  $\omega_d = -k_3 \delta - k_4 \delta^{m/n} - k_4 \delta^{p/q}$ . Then, their derivatives can be written as  $\dot{e}_1 = \dot{\delta} = \omega_0 \omega$ ,  $\dot{e}_2 = \dot{\omega} - \dot{\omega}_d$ , in which  $\omega$  is the virtual controller and  $\omega_d$  is the virtual stabilization function. Then, the derivative of  $e_1$  can be written as

$$\dot{e}_1 = \omega_0. \quad (16)$$

A Lyapunov candidate function  $V_2(t)$  for stability analysis is defined as follows:

$$V_2(t) = \frac{e_1^2}{2}. \quad (17)$$

Differentiating  $V_2(t)$  with respect to time, there is

$$\begin{aligned} \dot{V}_2(t) &= e_1 \dot{e}_1 \\ &= \omega_0 \left( e_1 e_2 - k_3 e_1^2 - k_4 e_1^{(m+n)/n} - k_4 e_1^{(p+q)/q} \right). \end{aligned} \quad (18)$$

From (18) we can know  $\dot{V}_2(t) \leq 0$  when  $e_2 = 0$ .

Step 2. Then, going one step ahead provides the candidate Lyapunov function  $V_3(t)$  as follows:

$$V_3(t) = V_2(t) + \frac{e_2^2}{2}. \quad (19)$$

Differentiating  $V_3(t)$  with respect to time yields

$$\begin{aligned} \dot{V}_3(t) &= \dot{V}_2(t) + e_2 \dot{e}_2 \\ &= \omega_0 \left( e_1 e_2 - k_3 e_1^2 - k_4 e_1^{(m+n)/n} - k_4 e_1^{(p+q)/q} \right) \\ &\quad + e_2 (\dot{\omega} - \dot{\omega}_d) \\ &= \omega_0 \left( -k_3 e_1^2 - k_4 e_1^{(m+n)/n} - k_4 e_1^{(p+q)/q} \right) \\ &\quad + e_2 \left[ \omega_0 e_1 + \frac{1}{T_{ab}} (m_t - D\omega - P_e) + u_\omega \right. \\ &\quad \left. + k_3 \omega_0 \omega + \frac{mk_4 \omega_0 \omega}{n} \cdot \delta^{(m-n)/n} \right. \\ &\quad \left. + \frac{pk_4 \omega_0 \omega}{q} \cdot \delta^{(p-q)/q} \right]. \end{aligned} \quad (20)$$

In order to achieve fixed-time stability for the  $V_3(t)$ , the actual fixed-time feedback control law is obtained as

$$\begin{aligned} u_\omega &= -\omega_0 \delta - \frac{1}{T_{ab}} (m_t - D\omega - P_e) - k_3 \omega_0 \omega - \frac{mk_4 \omega_0 \omega}{n} \delta^{(m-n)/n} \\ &\quad - \frac{pk_4 \omega_0 \omega}{q} \delta^{(p-q)/q} - k_3 e_2 - k_5 e_2^{m/n} - k_5 e_2^{p/q}. \end{aligned} \quad (21)$$

To bring  $u_\omega$  into  $\dot{V}_3(t)$  the following can be obtained:

$$\begin{aligned} \dot{V}_3(t) &= \omega_0 \left( -k_3 e_1^2 - k_4 e_1^{(m+n)/n} - k_4 e_1^{(p+q)/q} \right) \\ &\quad - k_3 e_2^2 - k_5 e_2^{(m+n)/n} - k_5 e_2^{(p+q)/q} \\ &\leq -k_4 \omega_0 e_1^{(m+n)/n} - k_5 e_2^{(m+n)/n} - k_4 \omega_0 e_1^{(p+q)/q} - k_5 e_2^{(p+q)/q} \\ &\leq -k_m \left( e_1^{(m+n)/n} + e_2^{(m+n)/n} \right) - k_m \left( e_1^{(p+q)/q} + e_2^{(p+q)/q} \right) \\ &= -k_m \left[ \left( \frac{e_1^2}{2} \right)^{(m+n)/2n} \left( \frac{1}{2} \right)^{-(m+n)/2n} \right. \\ &\quad \left. + \left( \frac{e_2^2}{2} \right)^{(m+n)/2n} \left( \frac{1}{2} \right)^{-(m+n)/2n} \right] \\ &\quad - k_m \left[ \left( \frac{e_1^2}{2} \right)^{(p+q)/2q} \left( \frac{1}{2} \right)^{-(p+q)/2q} \right. \\ &\quad \left. + \left( \frac{e_2^2}{2} \right)^{(p+q)/2q} \left( \frac{1}{2} \right)^{-(p+q)/2q} \right] \\ &= -k_m 2^{(m+n)/2n} \left[ \left( \frac{e_1^2}{2} \right)^{(m+n)/2n} + \left( \frac{e_2^2}{2} \right)^{(m+n)/2n} \right] \\ &\quad - k_m 2^{(p+q)/2q} \left[ \left( \frac{e_1^2}{2} \right)^{(p+q)/2q} + \left( \frac{e_2^2}{2} \right)^{(p+q)/2q} \right] \\ &\leq -k_m 2^{(m+n)/2n} V_3^{(m+n)/2n}(t) - 2k_m V_3^{(p+q)/2q}(t), \end{aligned} \quad (22)$$

TABLE 1: The system and controllers parameters.

Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
$D$	0.5	$E'_q$	1.35	$x'_d \Sigma$	1.15	$e_y$	1
$e$	0.7	$h$	2	$x_q \Sigma$	1.474	$T_{ab}$	8
$e_{qh}$	0.5	$T_y$	0.1	$T_r$	1	$V_s$	1
$\omega_0$	314	$\delta_0$	30	$x_L$	0.08	$k_1$	2.5
$k_2$	0.5	$k_3$	0.05	$k_4$	0.05	$k_5$	1
$m$	11	$n$	15	$p$	11	$q$	7

where  $k_m = \min(k_4 \omega_0, k_5)$ ,  $\alpha_2 = k_m 2^{(m+n)/2n}$ ,  $\beta_2 = 2k_m$ ,  $\phi_2 = (m+n)/2n$ , and  $\varphi_2 = (p+q)/2q$ . According to Lemma 1, we know that  $\dot{V}_3(t)$  is stable in fixed time.

$$t_2 = \frac{\pi q n}{k_m (pn - mq) \cdot 2^{(m-n)/4n}}, \quad (23)$$

which means  $V_3(t) = 0$  when  $t \geq t_2$ . It also means  $e_1 = 0$ ,  $e_2 = 0$ ,  $\delta = 0$ , and  $\omega = 0$ , and the system reaches steady state. To sum up, when  $t \geq t_3$ , the hydraulic turbine governing system is stable under the action of fixed-time feedback controllers  $u_\omega$  and  $u_y$ , where  $t_3 = t_1 + t_2$ . In other words, the system is stable in the fixed time.

#### 4. Simulation Results and Analyses

In this section, the simulation results are provided to verify the validity and effectiveness of the proposed fixed-time feedback control strategy. Based on some published papers, the value of system parameters are selected from [18] and the system parameters and controller parameters of this paper are shown in Table 1.

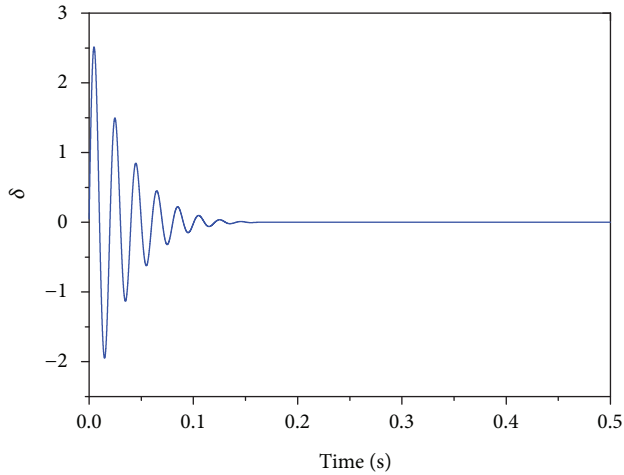
Figures 1(a)–1(c) display the transient responses of the system state variables  $\delta$ ,  $\omega$ , and  $y$  after the controllers  $u_\omega$  and  $u_y$  are applied to the hydraulic turbine governing system, respectively. From Figure 1, it can be seen that when the system is coupled with  $u_\omega$  and  $u_y$ , the system state variable  $y$  reaches steady state at 0.11 s, and the system state variables  $\delta$  and  $\omega$  reach steady state at 0.18 s, simultaneously. The convergence time of  $\delta$ ,  $\omega$ , and  $y$  is much smaller than the theoretical estimate  $t_3 = 4.634$  s, which is calculated by the values of the parameters of the fixed-time feedback controllers. The simulation results show that the system can reach the steady state rapidly under the action of fixed-time feedback controllers  $u_\omega$  and  $u_y$ , and the control effect is achieved.

In this part, in order to prove the validity and superiority of the fixed-time feedback controllers proposed in this paper, we compared the control effect of the fixed-time feedback controllers and the feedback controllers. From Section 3, we know that the fixed-time feedback controllers are composed of two parts of the fixed-time control part and the feedback control part, and the feedback control part can also bring the hydraulic turbine governing system to a stable state in theory. The feedback controllers used for comparison are the feedback control part of the fixed-time feedback

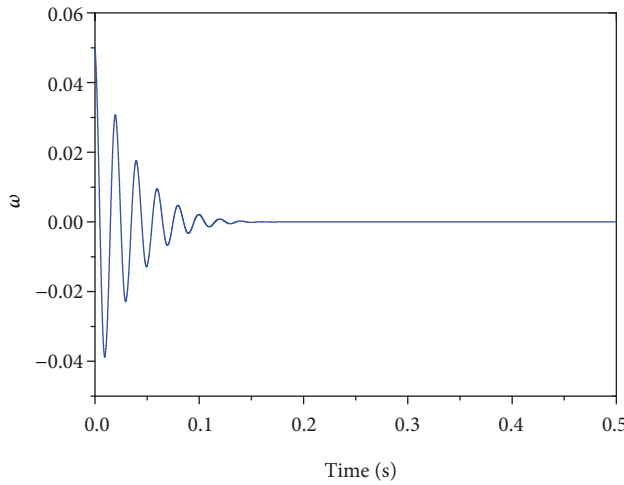
controllers. In order to make a fair comparison between fixed-time feedback control and feedback control, we select the same initial conditions and system parameters. The parameters of the feedback controllers are exactly the same as those of the feedback control part of the fixed-time feedback controllers and these parameters have been given in Table 1. Figures 2(a)–2(c) show the dynamic responses of the system state variables under the action of fixed-time feedback controllers and feedback controllers. From Figure 2, it can be found that the performance of the system state variables  $\delta$ ,  $\omega$ , and  $y$  with fixed-time feedback controllers is much better than the performance of the system state variables  $\delta$ ,  $\omega$ , and  $y$  with the feedback controllers. Under the action of fixed-time feedback controllers, the system state variables  $\delta$ ,  $\omega$ , and  $y$  are stable at 0.18 s, 0.18 s, and 0.11 s, respectively. And under the action of feedback controllers, the system state variables  $\delta$ ,  $\omega$ , and  $y$  are stable at 1.08 s, 1.08 s, and 0.25 s, respectively. The fixed-time controllers stabilize the nonlinear system faster than feedback controllers do. In other words, compared with the feedback control, the fixed-time feedback control has better control effect on the hydraulic turbine governing system.

In order to explore the effect of different initial conditions on system stability under the action of fixed-time feedback controllers, we compared the responses of three different initial conditions of the hydraulic turbine governing system. In this section, the initial values of the hydraulic turbine governing system are shown as follows:  $S1 = (0.02, 0.02, 0.02, 0.02, 0.02, 0.02)$ ,  $S2 = (0.05, 0.05, 0.05, 0.05, 0.05, 0.05)$ , and  $S3 = (0.08, 0.08, 0.08, 0.08, 0.08, 0.08)$ . From Figure 3, we can see that as the initial values increase, the convergence time gradually increases when the system is coupled with  $u_\omega$  and  $u_y$ , but they all do not exceed the theoretical estimate  $t_3 = 4.634$  s. And no matter how the initial value changes, the system achieves the ideal steady state. That is to say, the simulation results are consistent with the theoretical derivation and the actual situation.

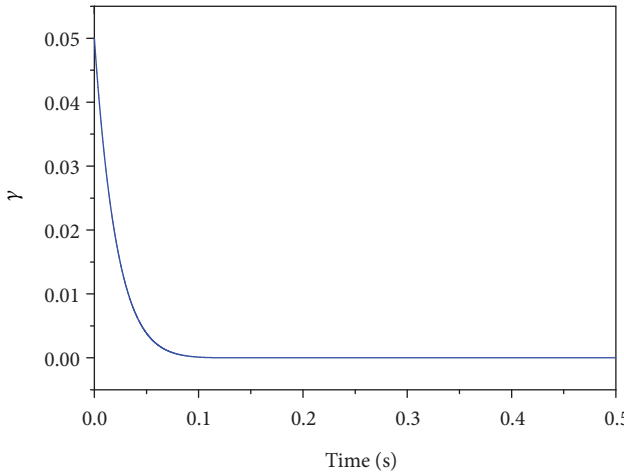
Figures 4(a)–4(c) show the dynamic responses of the system state variables when the system is subjected to external disturbances. In order to verify the robustness of the controlled system to disturbance, this paper gives  $\delta$  and  $y$  a disturbance in 0.5 s and 1 s, respectively. From Figure 4, it can be found that the system achieves steady state under the action of fixed-time feedback controllers before 0.5 s. When the system is disturbed in 0.5 s,  $\delta$  and  $\omega$  quickly



(a)



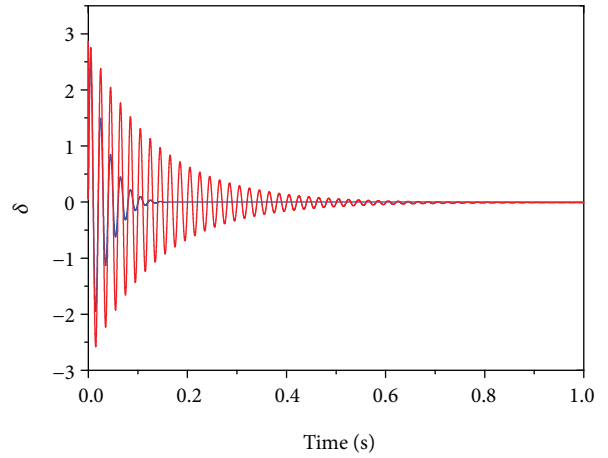
(b)



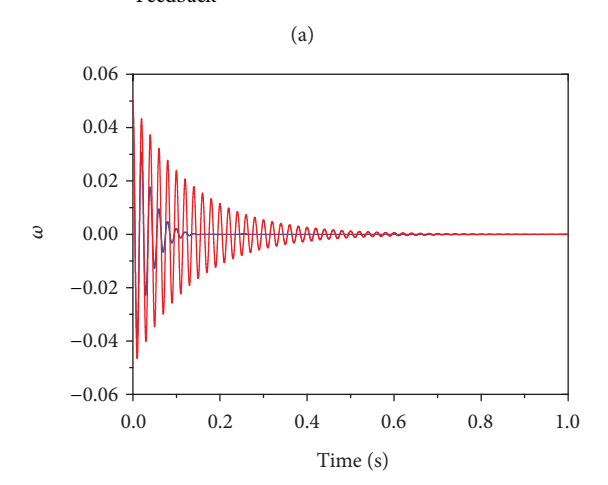
(c)

FIGURE 1: The response of the system variables  $\delta$ ,  $\omega$ , and  $\gamma$  with fixed-time feedback controllers.

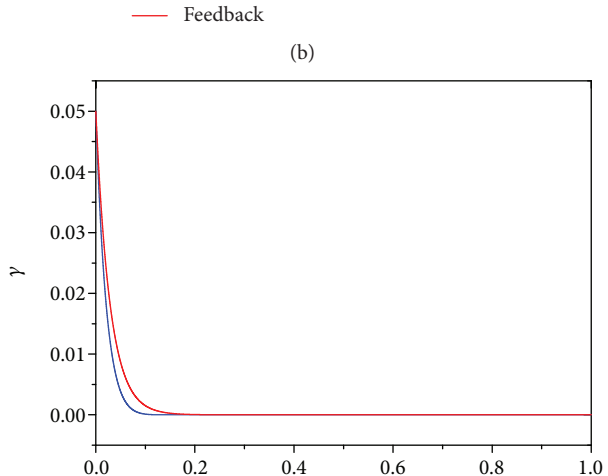
increase, whose maximum deviations reach to 2.51 and 0.05, respectively. But under the action of the controller  $u_\omega$ , the state of  $\gamma$  has not changed. As time goes on,  $\delta$  and  $\omega$  gradually



(a)

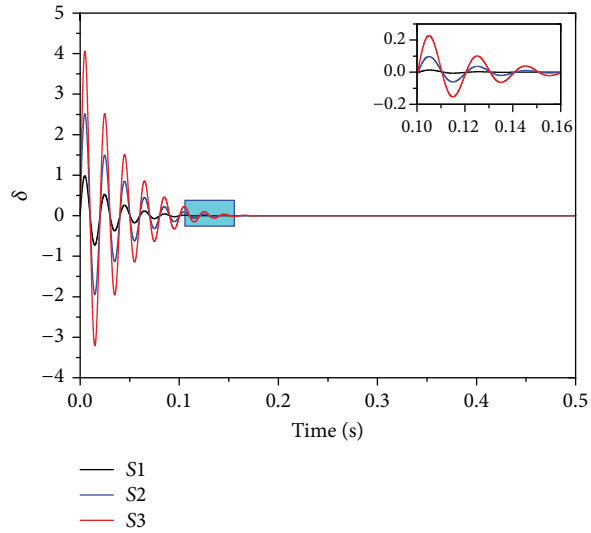


(b)

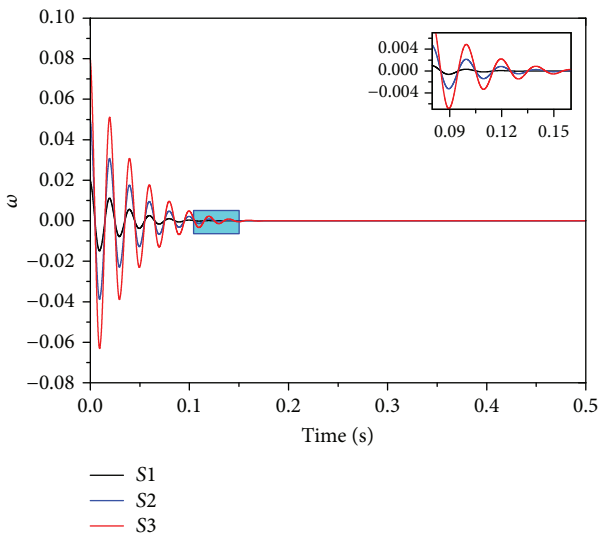


(c)

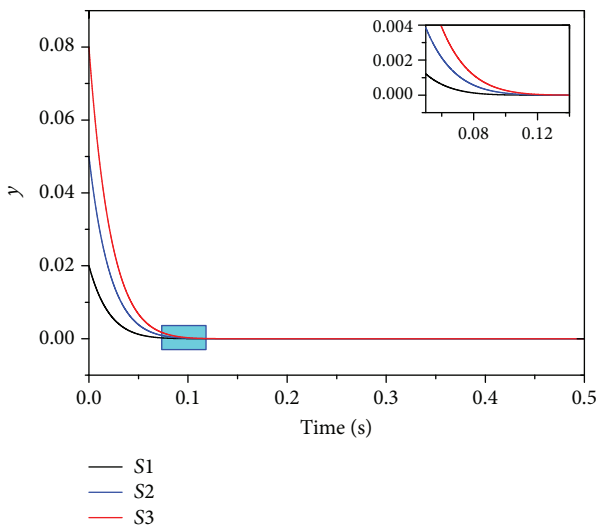
FIGURE 2: Comparison of converging speed of fixed-time feedback and feedback controllers.



(a)

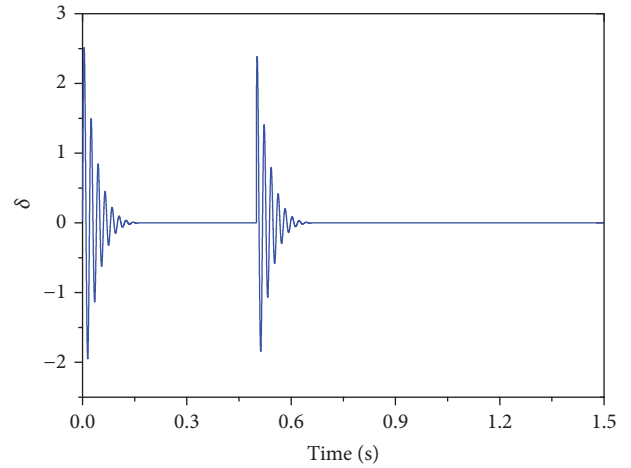


(b)

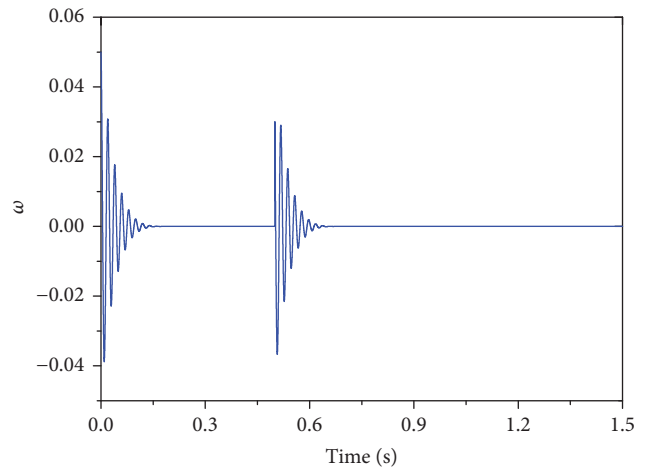


(c)

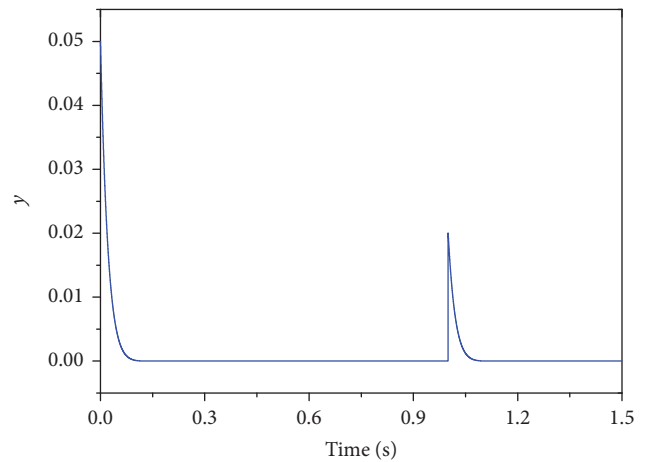
FIGURE 3: The response of the system with the different S.



(a)



(b)



(c)

FIGURE 4: The response of the system with the different disturbances.

reach the steady state, which indicate that the hydraulic turbine governing system remains in a stable state under the appropriate control of the fixed-time feedback controller  $u_\omega$ . When the system is disturbed in 1 s,  $y$  gradually decreased

from the disturbance and eventually stabilized to zero. There is no change in the state of  $\delta$  and  $\omega$  in the controlled system. Therefore, the simulation results are consistent with the theoretical derivation and show that the system has good robustness to external disturbances under the action of fixed-time feedback controllers.

## 5. Conclusions

The control problem of a nonlinear hydraulic turbine governing system is studied in this paper. In order to give the system better quality, the fixed-time feedback controllers were designed by combining fixed-time control and feedback control and compared with the feedback controllers. The designed controllers are without sign function, which means the chattering phenomenon will not appear in the system. The fixed-time feedback control inherits the advantages of the fixed-time control, so that the maximum convergence time of the system is not affected by the initial state of the system. The introduction of feedback control of the fixed-time feedback control makes the controlled system achieve a steady state faster. The fixed-time feedback controllers also give the system robustness to external disturbances. Finally, simulations were carried out in the presence of various situations to evaluate the effectiveness of the proposed fixed-time feedback controllers, where the results demonstrated the effectiveness and superiority of the proposed controllers.

## Nomenclature

$D$ :	Generator damping coefficient
$e$ :	Intermediate variable
$e_{qh}$ :	First-order partial derivative value of flow rate with respect to water head
$e_y$ :	First-order partial derivative value of torque with respect to wicket gate
$E'_q$ :	The transient internal voltage of armature
$h_\omega$ :	The characteristic coefficient of the pipeline
$m_t$ :	The deviation of the mechanical torque of the hydro-turbine
$P_e$ :	The electromagnetic power of generator
$T_y$ :	The engager relay time constant
$T_r$ :	The reflection time of the penstock
$T_{ab}$ :	The mechanical starting time
$x'_d$ :	The direct axis transient reactance
$x_q$ :	The quartered axis reactance
$x_T$ :	Short circuit reactance of the transformer
$x_L$ :	Reactance of the electric transmission line
$x'_{d\Sigma}$ :	Direct axis transient reactance
$x'_{q\Sigma}$ :	Quadrature axis reactance
$V_s$ :	The voltage of infinite bus
$\delta_0$ :	Initial generator rotor angle
$\omega_0$ :	Synchronous angular speed.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

## Authors' Contributions

Caoyuan Ma and Chuangzhen Liu conceived and designed the experiments. Chuangzhen Liu and Xuezi Zhang performed the experiments and analyzed the data. Chuangzhen Liu and Xuezi Zhang wrote the paper. Caoyuan Ma and Chuangzhen Liu contributed equally to this work.

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